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DISLOCATION THEORY OF THE FATIGUE OF METALS

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SUMMARY

A dislocation theory of fatigue failure for annealed solid solutions is presented. On the basis of this theory, an equation giving the dependence of the number of cycles for failure on the stress, the temperature, the material parameters, and the frequency is derived for uniformly stressed specimens. The equation is found to be in quantitative agreement with the data. Inasmuch as one material parameter is indicated to be temperature-dependent and its temperature dependence is not known, it is impossible to predict the temperature dependence of the number of cycles for failure. A predicted quantitative correlation between fatigue and creep is found to exist, which suggests the practical possibility of obtaining fatigue data for annealed solid solutions and elements from steady-state creep-rate data for these materials. As a result of this investigation, a modification of the equation for the steady-state creep rate previously developed on the basis of dislocation theory is suggested. Additional data are required to verify completely the dislocation theory of fatigue.

INTRODUCTION

The failure of materials under an oscillating stress, the maximum value of which is lower than that required to cause failure in a static tensile test, is termed "fatigue failure." This type of failure is the limiting factor in the design and the operation of many rotating parts. In particular, fatigue strength has become a limiting factor in the development of the gas turbine for aircraft use. Turbine buckets, for example, have in many cases failed as a result of fatigue.

Two methods of eliminating the problem of fatigue failure exist: (1) The design of the parts can be changed to minimize the effect of oscillating stresses; and (2) the resistance to fatigue failure of the materials used can be increased. A phase of the second solution, based on obtaining an understanding of the fatigue phenomena, is discussed in this report.

Although many experimental investigations of fatigue have been made, few have been concerned with basic considerations. The work of Gough and his collaborators (references 1 and 2), however, is outstanding in the field of fundamental fatigue studies. This work has shown that the phenomena of plastic deformation (slip) and fatigue are closely related. Some theories have been reported (references 3 and 4) that relate plastic deformation to fatigue failure. Because these theories are not of a fundamental nature, however, they have not yielded basic knowledge of the physical properties of materials that affect fatigue.

With the development of the theory of dislocations, the understanding of the phenomena of plastic deformation has greatly advanced (references 5 to 8). Inasmuch as plastic deformation and fatigue have been shown to be related, an investigation of fatigue based on the theory of dislocations was undertaken and is presented herein. In this investigation a physical model, based on dislocation theory, was devised to account for crack growth. This model was then used as a means of obtaining an equation relating the number of cycles for failure to the appropriate variables. The equation so obtained was then subjected to various experimental checks. This investigation is part of a program being conducted at the NACA Cleveland laboratory to evaluate the physical properties of heat-resisting alloys in terms of physical constants that are easily measurable and to make possible the synthesis of compositions and structures of heat-resisting alloys better than those currently used.

SYMBOLS

The following symbols are used in this report:

a constant

c proportionality constant relating τ_a to $\sigma_m/2$

d_1 distance between atoms in slip direction, centimeters

f fraction, experimental value = 0.374 (reference 8)

ΔF_g free energy of activation involved in generation of a dislocation, ergs per molecule

h Planck's constant, 6.62×10^{-27} erg seconds

ΔH_g heat of activation per molecule involved in generation of a dislocation, ergs

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k Boltzmann's constant, 1.38×10^{-16} ergs per molecule per $^{\circ}K$

L distance between imperfections in single crystal, of order of 1 micron

M amount of crack growth per crack source necessary for failure measured in units of number of interatomic spacings

N number of cycles for failure

P period of cycle of stress, seconds

q stress-concentration factor

q' cq

R_d rate of generation of positive or negative dislocations at stress concentration of internal surface in crystalline material, number of dislocations per second

R_g net rate of growth per crack, number of atomic spacings per second

ΔS_g entropy of activation per molecule involved in generation of dislocation, ergs per $^{\circ}K$

T absolute temperature, $^{\circ}K$

t time, seconds

u' shear rate, 1 per second

u_1 antilog of intercept of plot of $\log u'$ against σ at $\sigma = 0$

V atomic volume, cubic centimeters

x ratio of d_1 to interplanar spacing of slip planes, $\left(\sqrt{\frac{3}{2}}\right.$ for face-centered and body-centered cubic lattices)

y mathematical variable

θ ωt

σ tensile stress, dynes per square centimeter

σ_m maximum tensile or compressive stress of cycle, dynes per square centimeter

τ resolved shear stress, dynes per square centimeter

τ_a average resolved shear stress in polycrystalline specimen operating for failure, dynes per square centimeter 777

τ_i resolved shear stress due to internal source, dynes per square centimeter

τ_m resolved maximum shear stress corresponding to σ_m, dynes per square centimeter

τ_s average shear stress at sources of dislocations, dynes per square centimeter

ω frequency of cycles of stress application, 1 per second

CRACK MECHANISM

Fatigue failure is usually associated with the propagation of a crack. This fatigue crack must be formed either as a result of internal tensile stresses that exceed the tensile strength of the material or from pre-existing submicroscopic cracks that grow under the influence of the cyclic stress. The assumption will be made that such submicroscopic cracks do exist. It is probable that some of the sources of stress concentrations in metals are in the form of submicroscopic cracks that occur at the boundaries of mosaic blocks (reference 6).

A theory that would explain the growth of these submicroscopic cracks as a result of the continual reversal of stress might provide a basis for the development of a quantitative theory of fatigue. The analysis of the mechanism of crack formation developed herein is predicated on the dislocation theory discussed in references 5, 6, 8, and 9. A short résumé of the dislocation theory follows:

A dislocation consists of a stable arrangement of atoms such that, in a region of a few atomic distances, $n + 1$ atoms in the slip direction face n atoms across the slip plane. When the $n + 1$ atoms are above the n atoms, the configuration is called a positive dislocation; when the $n + 1$ atoms are below, it is called a negative dislocation. The net result of a positive dislocation moving completely through a specimen from left to right, or a negative dislocation moving completely through the specimen from right to left, is a translation of the material above the plane between the $n + 1$ atoms and the n atoms, with respect to the material below this plane, by one atomic distance to the right. If the positive dislocation moves from right to left or the negative dislocation moves from left to right, the opposite translation takes place.

The most probable sources of generation of dislocations are stress concentrations. Such stress concentrations occur at the ends of cracks. For example, an ellipsoidal crack shown in cross section in figure 1, has two points of stress concentration (A and B in fig. 1), which exceed the stress concentration at any other point along the crack circumference (reference 10). Dislocations will usually be generated at these stress concentrations upon the application of a shear stress.

In the development of a theory for the growth of submicroscopic cracks, it was found that as a result of making certain assumptions about the physical model, it was possible to obtain an equation that was in agreement with the available data. Although each of these assumptions may have a physical basis, such a basis was not immediately evident. The development of the theory that follows therefore presents these assumptions without any attempt to justify them physically.

It is assumed that cracks similar to the one shown diagrammatically in figure 1 exist in such a manner that points A and B act as sources of generation of dislocations. It is further assumed that only positive dislocations can be generated at point A and only negative dislocations can be generated at point B. If the line joining A and B is at a small angle to the slip plane and if, at point A, the generated positive dislocations move to the right and, at point B, the generated negative dislocations move to the left as a result of the given shear stress, the crack will grow. An intermediate position of the right side of the crack after the crack has grown to some extent is shown by the dashed line in figure 1.

If all the dislocations that were generated at A and B during one half-cycle of stress had returned to their respective sources during the next half-cycle of stress, the crack would not have grown at the end of a complete cycle of reversed stressing. In order for the crack to grow, some of the dislocations generated at the crack sources during the growth part of the stress cycle therefore must either disappear from the specimen or reach a point from which their return to the source is prevented. No dislocations generated at A and B in the second half-cycle of stress can enter the crystal lattice because, according to the assumption, only positive dislocations are generated at A and only negative dislocations are generated at B, and the applied stress during this half-cycle of stress would make any dislocations generated at these sources return to their respective sources. This assumption does not mean, however, that other sources at which none of the dislocations there generated move into the crystal lattice during the first half-cycle will not generate dislocations that move into the crystal lattice in the last half-cycle.

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If one submicroscopic crack were to account for the failure of a macroscopic specimen, it would appear that more than the observed amount of plastic flow would have to occur. Hence, it seems evident that if failure were to occur as the result of the growth of submicroscopic cracks, it would take place because many cracks contribute to the failure. A simple configuration of cracks of the type illustrated in figure 1 that can lead to fatigue failure with little plastic flow is shown in figure 2.

This configuration, after all the cracks have grown to a certain extent, would appear as shown in figure 3. In order that the cracks may grow in this configuration it becomes necessary, as before, to assume that at the points A only positive dislocations are generated and at the points B only negative dislocations are generated. The result of positive dislocations moving to the left and negative dislocations simultaneously moving to the right during a cycle of stress favorable for this motion, is that relative translations of the lattice occur which yield larger cracks than existed prior to the translations. Inasmuch as these dislocations disappeared from the lattice at the points C and D in figure 2, the application of a stress opposite to the previous stress would not cause any relative translations of the lattice between the regions separated by the slip planes A-C and B-D, unless positive and negative dislocations were generated at C and D, respectively. It is assumed that no dislocations are generated at points C and D. Each crack in the configuration will therefore grow as long as positive dislocations generated at A and negative dislocations generated at B disappear from the lattice at C and D, respectively.

In the foregoing discussion, it was assumed that each crack extends completely through the specimen in the direction perpendicular to the plane of the figure. Because the cracks were assumed to exist in the mosaic-block boundaries, however, it seems probable that the crack width in the direction considered is not more than a few mosaic-block lengths, that is, about 10^{-4} centimeters. In order to account for the growth of short cracks it thus becomes necessary to assume that such short dislocations may exist and also that relative slip between mosaic blocks may easily occur. Seitz and Read in considering the phenomena of slip with respect to dislocations found that these assumptions would help explain the experimental slip phenomena (reference 6, pt. II).

One of the consequences of the crack-growth mechanism just described is that the cracks will begin to grow along slip planes. This phenomenon has been observed by a number of experimenters (for example, references 2 and 11) on polycrystalline as well as single-crystal specimens. It was noticed that failure began to occur along the slip plane and then proceeded in the direction normal to the maximum tensile stress.

NUMBER OF CYCLES FOR FAILURE

The general equation for the rate of generation of dislocations R_d of one type at a stress concentration of an internal surface in a crystalline material is given in reference 8. The equation, for large values of $\frac{2qVxf\tau}{kT}$, can be written as

$$R_d = \frac{kT}{h} e^{\left(\frac{-\Delta F_g}{kT} + \frac{2qVxf\tau}{kT}\right)} \tag{1}$$

The net rate of growth per crack R_g is the rate of generation of dislocations during the growth cycle:

$$R_g = R_d = \frac{kT}{h} e^{\left(\frac{-\Delta F_g}{kT} + \frac{2qVxf\tau}{kT}\right)} \tag{2}$$

The stress at a generating source may consist of the applied stress and any internal stress that may be present. As a result,

$$\tau_s = q\tau = q(\tau_m \sin\theta + \tau_i) \tag{3}$$

If it is assumed that the amount of crack growth per crack source required for failure M is a constant for all specimens and materials, that the stress-concentration values are the same for every crack associated with failure, and that these stress-concentration values do not change with the number of cycles of stressing, then the dependence of the number of cycles for failure on the maximum resolved shear stress in the slip plane and direction considered is

$$M = N \int_0^{\frac{P}{2}} R_g \, dt \tag{4}$$

because R_g is independent of N.

Substituting for R_g in equation (4) yields

$$M = N \frac{kT}{h} \left(e^{\frac{-\Delta F_g}{kT}} \right) \int_0^{\frac{P}{2}} e^{\frac{2qVxf\tau}{kT}} \, dt \tag{5}$$

For annealed metals, the internal stress τ_i is approximately zero. Thus,

$$\tau = \tau_m \sin\theta$$

Changing the variable of integration in equation (5) yields

$$M = \frac{NkT}{2\pi\omega h} \left(e^{\frac{-\Delta F_g}{kT}} \right) \int_0^{\pi} e^{\frac{2qVxf\tau_m \sin\theta}{kT}} \, d\theta \tag{6}$$

In appendix A it is shown that the integral $\int_0^{\pi} e^{ay \sin\theta} \, d\theta$ can be approximated for large values of ay by the factor $e^{0.422ay}$. When this approximation is used in equation (6) and equation (6) is solved for N

$$N = \frac{2\pi\omega hM}{kT} e^{\left(\frac{\Delta F_g}{kT} - \frac{0.844qVxf\tau_m}{kT} \right)} \tag{7}$$

In order to apply equation (7) to polycrystalline specimens, some assumptions must be made as to the method of fatigue failure of polycrystals. It seems probable that the orientations of the crystals comprising the fatigue-failed regions of a polycrystal are random. The maximum resolved maximum shear stress, if each of these crystals was considered apart from the others, would therefore vary from some minimum (not zero for face-centered cubic or body-centered cubic lattices) to the maximum $\frac{\sigma_m}{2}$. Inasmuch as these crystals are not separated by stress-free regions, internal stresses would be set up between the differently oriented crystals. As a result, steady-state slip would occur at some stress intermediate to the minimum and maximum value of the maximum resolved maximum shear stress. The value of this average resolved shear stress should then be substituted in equation (7) for polycrystalline specimens. If the average resolved shear stress is related to the maximum resolved shear stress by the equation $\tau_a = c\frac{\sigma_m}{2}$ and if $q' = cq$, then,

$$\log N = \log\left(\frac{2\pi\omega hM}{kT}\right) + \frac{\Delta F_g}{2.3kT} - \frac{0.422q'Vxf\sigma_m}{2.3kT} \tag{8}$$

It should be pointed out that equation (8) applies only for stresses sufficiently above the endurance limit (where the $\log N - \sigma_m$ function is linear), because the conditions that determine the endurance limit have not been considered in the development of the theory. Also, because the assumption was made that σ_m is constant over the cross section of the specimen, equation (8) will apply to axial-stress-type loading only. The cases of bending-type stresses and torsional stresses are theoretically capable of being treated in a manner similar to the case of uniform axial stress; however, no attempt has been made to evaluate equations of the type of equation (8) for these cases.

The intercept of the straight line obtained by plotting $\log N$ against σ_m at $\sigma_m = 0$, according to equation (8), is equal to

$$\log\left(\frac{2\pi\omega hM}{kT}\right) + \frac{\Delta F_g}{2.3kT}$$

If all the parameters in this expression are known then it should be possible to obtain a calculated value of the intercept to compare with the experimental value. The parameters ω and T are known from the experimental conditions. The parameter M is of the order of magnitude of a mosaic block length, measured in units of interatomic spacings, i.e., about 10^4. The parameters k and h are known constants. The only factor which remains to be determined is ΔF_g. Now, reference 8 presents a method of calculating ΔF_g using creep data. As shown in appendix B, however, because an assumption made in reference 8 is incorrect, ΔF_g cannot be completely determined from creep data. The analysis presented in appendix B does indicate, however, that a set of fatigue data and creep data for the same material can be used to calculate a value of the unknown factor. If the value of this factor is assumed to be independent of the material variable, then it can be used to help calculate values of ΔF_g using creep data. A table of such calculated values using data for copper, shown in figure 4(a), to evaluate the unknown factor is presented in appendix B.

A check of equation (8) can now be made by comparing the experimental value of the intercept of the straight line obtained by plotting $\log N$ against σ_m with the value obtained from calculation using the term derived from equation (8). Data for Armco iron, obtained from reference 12 and plotted in figure 4(b), can be used to make the suggested comparison of experimental and calculated values of the intercept being considered. The experimental value of the intercept determined by extrapolating the best straight line through the points of figure 4(b) is equal to 22. The approximate uncertainty in this value is ± 2. The calculated value is

$$\log\left(\frac{2\pi\omega hM}{kT}\right) + \frac{\Delta F_g}{2.3kT} = \log\left(\frac{2\pi \times \frac{2200}{60} \times 6.62 \times 10^{-27} \times 10^4}{1.38 \times 10^{-16} \times 300}\right)$$

$$+ \frac{2.92 \times 10^{-12}}{2.3 \times 1.38 \times 10^{-16} \times 300} = 24.2$$

where $\omega = 2200$ cycles per minute and $T = 300^\circ$ K, as given by reference 12. The agreement between the predicted value of 24.2 and the observed value of 22 ± 2 tends to verify equation (8).

An additional check of the validity of equation (8) could be made if the value of the stress-concentration factor q were known.

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Because the value of q is unknown, the experimental value of the slope of the linear portion of the plot of $\log N$ against σ_m can be used to obtain a value of q'. This calculation is:

$$q' = \frac{2.3\ kT \times (\text{experimental slope, sq cm/dynes})}{0.422 \times V \times x \times f}$$

$$= \frac{2.3 \times 1.38 \times 300 \times 10^{-16}\left(\dfrac{1}{0.755 \times 2240 \times 6.9 \times 10^{4}}\right)}{0.422 \times 11.7 \times 10^{-24} \times 1.23 \times 0.374}$$

$$= 360$$

A similar procedure was followed to obtain a value of q' for annealed copper using the line shown in figure 4(a) to define the slope. A value of $q' = 390$ was obtained.

The fatigue theory can be further experimentally checked. If equation (8) is solved for the parameter $V\sigma_m$, the following equation is obtained:

$$V\sigma_m = U + W\ \Delta F_g \tag{9}$$

where

$$U = \left(\frac{2.3kT}{0.422q'xf}\right)\left[\log\left(\frac{2\pi\omega hM}{kT}\right) - \log N\right]$$

$$W = \frac{1}{0.422q'xf}$$

Data to check equation (9) were obtained from a number of sources. The details concerning the sources and precision of data for ΔF_g are contained in appendix B.

Because the data for the value of σ_m that corresponds to failure in 3.3×10^{6} cycles was difficult to obtain accurately, the following table lists, wherever possible, a number of sources for a given material:

Material	σ_m, dynes/sq cm	Reference
Aluminum	3.5×10^8	20
[a]Silver	6.2×10^8	24
Copper	8.2×10^8 8.65×10^8	12 25
Armco Iron	1.74×10^9 1.90×10^9	12 14
Nickel	2.06×10^9 2.98×10^9	26 25

[a]Data obtained from test applying torsional stresses to a single crystal. Because the corrections involved are in opposite directions, the value given is a fair approximation.

Inasmuch as different sources yielded different values of the stress corresponding to a certain number of cycles for failure for a given material and, also, because the values of ΔF_g were obtained by extrapolation, it was decided to plot the data as regions of approximately equal probability corresponding to each material. The data are therefore plotted as rectangles or lines in figure 5. The data in this figure correspond to failure in approximately 3.3×10^6 cycles; the frequency used to calculate U is 2200 cycles per minute; the temperature is 300° K.

The theoretical curve corresponding to these data is, according to equation (9), a straight line of slope W and intercept U. The value of q' was considered to be 390, as previously found for annealed copper. Because all the other factors involved in the calculation of U and W are known, it is possible to calculate the theoretical values of U and W. The calculated values of U and W are -157×10^{-16} ergs per molecule and 0.0132, respectively. The theoretical curve corresponding to these values of U and W is plotted as the straight line in figure 5; it is apparent that the theoretical line is in agreement with the data. It should be noted that figure 5 represents a correlation between creep and fatigue, inasmuch as the ordinate is a parameter determined from fatigue data and the abscissa is a parameter determined from creep data.

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Two other independent checks of the theory are possible: The first is a check of the temperature dependence of the number of cycles for failure; the second is based on the dependence of the number of cycles for failure on frequency.

The assumption is made that q is independent of temperature. If equation (8) is solved for σ_m, the following equation is obtained:

$$\sigma_m = A + B\ (C - \log T)\ T \tag{10}$$

where

$$A = \frac{\Delta H_g}{0.422q'Vxf}$$

$$B = \frac{2.3k}{0.422q'Vxf}$$

$$C = \log\left(\frac{2\pi\omega hM}{kN}\right) - \frac{\Delta S_g}{2.3k}$$

Data for Armco iron obtained from reference 14 is shown in figure 6. A cross plot of these data (temperatures in $^{\circ}K$) is shown in figure 7, which also gives the theoretical curve corresponding to this experimental plot. Data used to compute the theoretical values were obtained as follows: values of $\Delta H_g/V$ correspond to values of A/V from figure 6 of reference 8; values of ΔS_g were taken from the values listed in the table given on page 16 of reference 8, corrected to account for the factor determined in appendix B; the value of q' was taken as 390; the frequency ω was taken as 40 cycles per second, as reported in reference 14.

A study of figure 7 indicates that the theoretical curve yields too small a temperature variation of σ_m. It would therefore appear that equation (10) does not agree with the experimental data. Two explanations of these results are possible. The first explanation is that the data may be in error. The large scatter for the data corresponding to 500° C, as shown in figure 6, indicates that this explanation may be true. Furthermore, it is known that meaningful temperature studies on direct-stress specimens are difficult to perform because it is easy for the specimen to become misalined owing to the use of long specimen holders. The misalinement may then be a function of temperature and yield an apparent temperature effect, as shown in figure 7.

The second and more probable explanation is that the stress concentrations associated with the cracks in the mosaic-block boundaries are temperature-dependent and, hence, the assumption made in plotting the theoretical curve in figure 7 is not valid. In this case it becomes difficult to predict the temperature dependence of σ_m, inasmuch as no apparent basis exists for determining the dependence of the stress-concentration factor on temperature. 777

A final check of the dislocation theory of fatigue can be obtained by determining whether the dependence of the number of cycles for failure on the frequency, as given by equation (8), is experimentally obtained. It is indicated in equation (8) that the $\log N$ would be linearly related to the $\log \omega$ at constant maximum stress and temperature. The experiments reported in reference 15 yield results that verify the prediction qualitatively. In reference 15, it is shown that direct-stress experiments performed at 7000 cycles per minute required a greater number of cycles for failure than tests at 1200 cycles per minute. The tests at 1200 cycles per minute, however, were performed on a different type machine than the tests at 7000 cycles per minute; therefore the results would not be expected to agree quantitatively. The increase in $\log N$ was nevertheless sufficiently large that it is improbable that the difference in machines could be solely responsible for the effect noticed. Reference 16 indicates that no frequency effect occurs at low values of frequency. An examination of the data, however, reveals that many more data would be required in order to be able to draw any conclusion in regard to this effect.

EVALUATION OF RESULTS

Sufficient data to verify equation (8) completely are not available. Inasmuch as this equation is in quantitative agreement with available data, further experimentation designed to verify the dislocation theory of fatigue is justified.

The theory was developed for annealed elements or solid solutions; the data used to check the theory were obtained only from annealed elements; It seems reasonable to believe, however, that data for annealed solid solutions would also be in agreement with the theory.

It appears improbable that equation (8) would apply to the case of precipitation-hardened or strain-hardened materials. Certain modifications would be necessary, such as the introduction of an internal-stress term. This subject is sufficiently complex to justify an independent investigation.

It should be mentioned that the uncertainty in the data appears sufficiently large to allow reasonable doubt in the constancy of the stress-concentration factor q for all annealed elements and solid solutions. The possibility exists that an experimental plot of $q'V\sigma_m$ against ΔF_g would also be linear; however, the data are insufficient to resolve this question.

It has been experimentally observed that the number of cycles for failure at some given stress and temperature obeys a statistical distribution (reference 17). An explanation of this phenomenon on the basis of equation (8) would probably require that one or more of the following factors, M, ΔF_g, and q assume a range of values for a given material. It seems reasonable that the factors M and q may vary from specimen to specimen inasmuch as these quantities are probably dependent upon manufacturing variables such as casting, drawing, and heat-treating. It is thus possible to account for the scatter of data normally obtained in fatigue tests on the basis of the dislocation theory.

It should be noted that figure 5, which helps substantiate equation (8), also establishes a correlation between fatigue and creep as predicted by the theory and proves that fatigue failure depends upon plastic deformation. With the assumption of a constant value of q at room temperature, the prediction of fatigue data from a knowledge of creep data for annealed elements and solid solutions is made possible for practical purposes.

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APPENDIX A

EVALUATION OF DEFINITE INTEGRAL

The definite integral

$$\int_0^\pi e^{ay \sin\theta}\, d\theta$$

was evaluated for a series of values of ay by numerical integration. The value of a was set equal to 10.56×10^{-8} to correspond approximately to the expected values of the physical quantity it represented. A plot of the log of the integral against y is shown in figure 8. For values of $y > 10^8$, it appears that the equation of the curve is

$$\log \int_0^\pi e^{10.56 \times 10^{-8} y \sin\theta}\, d\theta = by$$

where b is the slope of the line in figure 8, 4.44×10^{-8}. As a result,

$$\int_0^\pi e^{10.56 \times 10^{-8} y \sin\theta}\, d\theta = e^{by} = e^{\left(\frac{b}{a}\right) ay}$$

$$= e^{0.422ay}$$

APPENDIX B

EVALUATION OF ΔF_G

The equation

$$u' = 10^{-4} \frac{kT}{h} e^{\frac{-\Delta F_g}{kT} + \frac{\beta\tau}{kT}}$$

obtained from reference 8 was first used to calculate values of ΔF_g from steady-state creep-rate data. This calculation was accomplished by plotting $\log u'$ against σ, setting the experimental value of the intercept at $\sigma = 0$ equal to

$$\left(-4 + \log \frac{kT}{h} - \frac{\Delta F_g}{2.3kT}\right)$$

and solving for ΔF_g, because all the other parameters are known in the expression for the intercept. When these values of ΔF_g were used to calculate the intercepts of the curves of $\log N$ plotted against σ_m and $Ѵ_m$ against ΔF_g, it was found that the calculated intercepts exceeded the experimental intercepts by a factor that varied between 1×10^{-13} to 3×10^{-13} ergs per molecule in the value of ΔF_g. In attempting to find an explanation for this discrepancy, it was determined that an assumption made in reference 8 regarding the value and constancy of the quantity 10^{-4} in the preceding equation was erroneous. The value of 10^{-4} was used as an approximation for $\frac{d_1}{L}$. In the derivation of the foregoing equation, the value of 10^{-4} was based on an assumption that one source of generation of dislocations exists per mosaic block. Thus, according to this assumption the strain due to the passage of one dislocation through a mosaic block is $\frac{d_1}{L} \cong \frac{3 \times 10^{-8}}{3 \times 10^{-4}} \cong 10^{-4}$. Evidence exists, however, for single crystals, which indicates that the spacings between slip planes may be many multiples greater than 3×10^{-4}. For example, figure 9 taken from reference 18, indicates that the spacing may be as large as 0.3 centimeter and figure 10 indicates that $1/L$ (number of slip planes/cm) may vary between 0 and 10,000. Thus it appears that the assumption that L is a constant for polycrystals of magnitude 3×10^{-4} is probably incorrect. It remains to be shown, however, how L may be calculated.

Without a physical model of the mechanism of strain in a polycrystal, it seems that the simplest assumption is to leave the factor $\frac{d_1}{L}$ as an unknown constant 10^C. The factor C can then be determined from a set of fatigue data and creep data for the same material as follows: From equation (8), it is evident that the intercept of the straight line obtained by plotting $\log N$ against σ_m, at $\sigma_m = 0$ is

$$\log\left(\frac{2\pi\omega hM}{kT}\right) + \frac{\Delta F_g}{2.3kT}$$

The data for annealed copper shown in figure 4(a) will be used in this calculation. The data for dry purified air were used to locate the slope of the line taken through the points for the tests in vacuum. The intercept of this line was found to equal about 15. Hence,

$$\Delta F_g = \left[15 - \log\left(\frac{2\pi \times \frac{2200}{60} \times 6.62 \times 10^{-27} \times 10^4}{1.38 \times 10^{-16} \times 300}\right)\right] 2.3 \times 1.38 \times 10^{-16} \times 300$$

$$= 2.01 \times 10^{-12} \text{ ergs/molecule}$$

where $\omega = 2200$ cycles per minute and $T = 300^\circ$ K, as given by reference 12.

From the creep equation,

$$\log u' = C + \log\frac{kT}{h} - \frac{\Delta F_g}{2.3kT} + \frac{\beta\tau}{kT}$$

it is apparent that the intercept $\log u_i$ of the straight line obtained by plotting $\log u'$ against τ at $\tau = 0$ is

$$\log u_i = C + \log\frac{kT}{h} - \frac{\Delta F_g}{2.3kT}$$

An estimate of $\log u_1$ at 300° K can be obtained from data given at other temperatures by plotting $(T \log \frac{u_1}{T})$ against $\frac{1}{T}$, because ΔF_g is linearly dependent on T, and by extrapolating the best straight line thus obtained to $T = 300°$ K. Using the data of references 22 and 23, an average value of -15 was obtained for $\log u_1$. Substituting this value for $\log u_1$, and $\Delta F_g = 2.01 \times 10^{-12}$ in the preceding equation and solving for C yields

$$C = -6.7$$

For the purpose of obtaining values of ΔF_g from creep data for other materials, C was assumed to be a constant. Using this value of C, values of ΔF_g were obtained from creep data and are given in the following table together with a list of the sources of the respective data.

Material	ΔF_g, ergs/molecule	Reference
Aluminum	$1.57 - 1.70 \times 10^{-12}$	18, 21
Silver	$1.85 - 2.26 \times 10^{-12}$	18
Copper	$1.94 - 2.12 \times 10^{-12}$	22, 23
Armco Iron	$2.74 - 3.09 \times 10^{-12}$	13
Nickel	$3.13 - 3.63 \times 10^{-12}$	21

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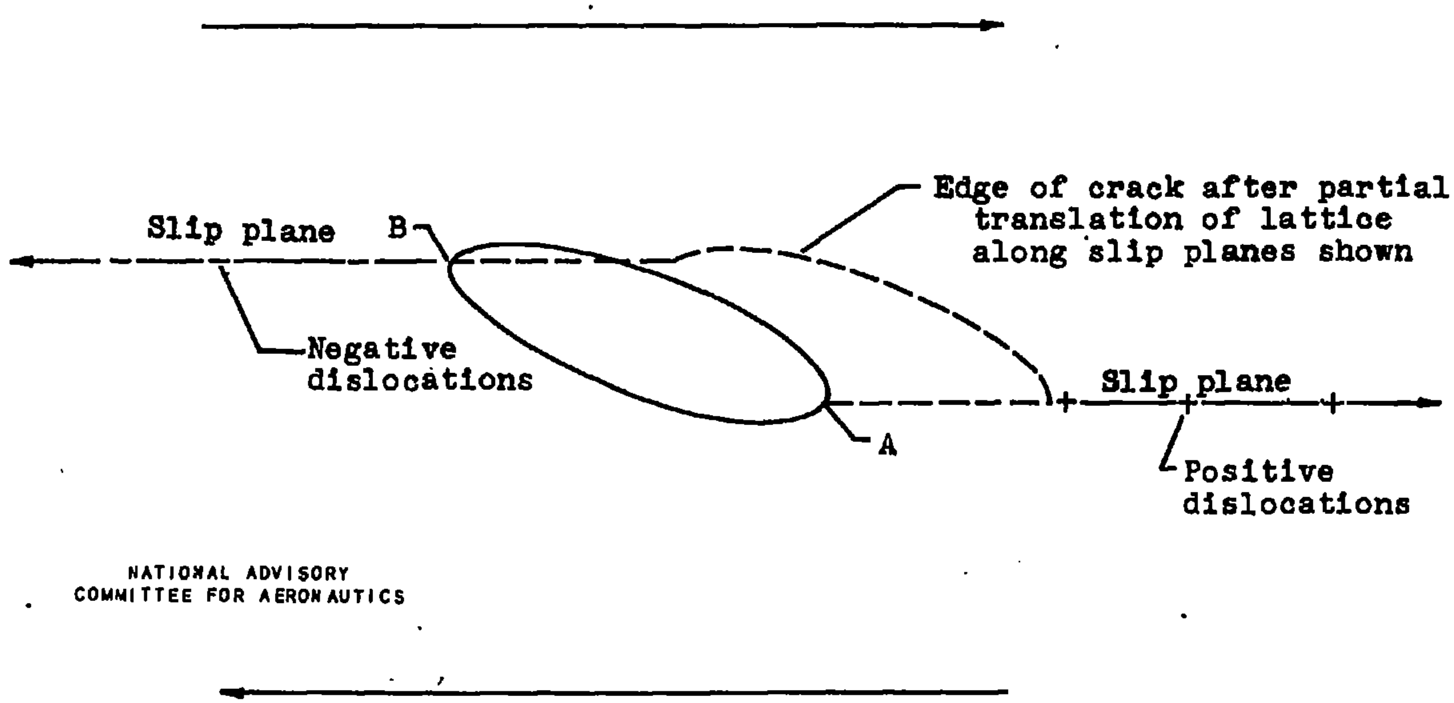

Figure 1. - Single crack generating source of dislocations.

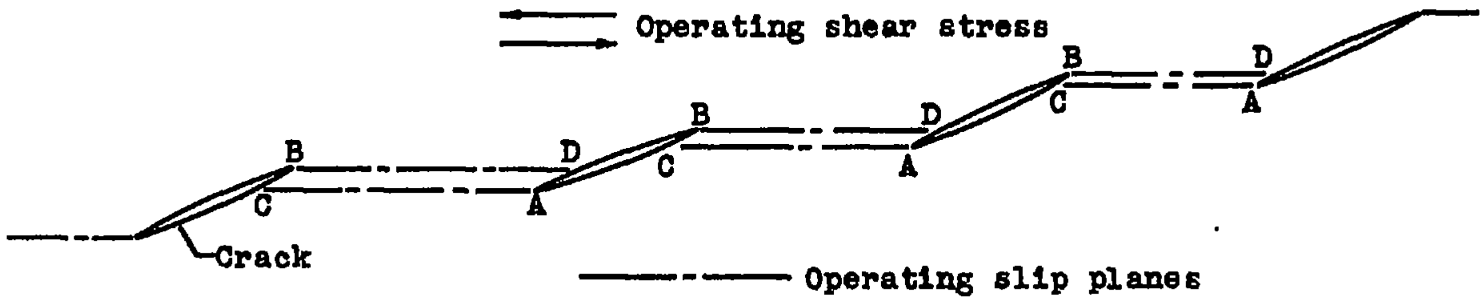

Figure 2. - Submicroscopic crack configuration that may lead to fatigue failure.

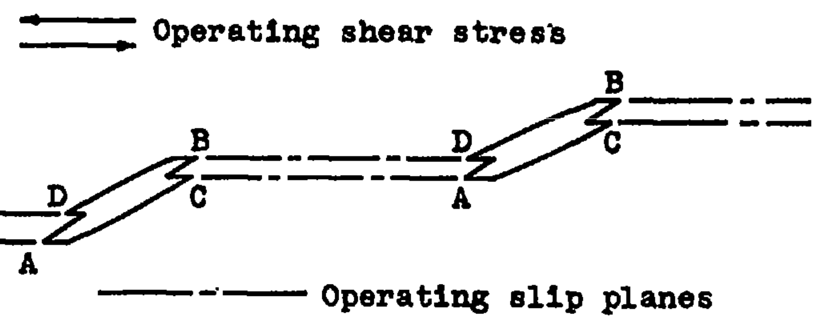

Figure 3. - Submicroscopic crack configuration after partial slip has occurred leading to growth of the cracks.

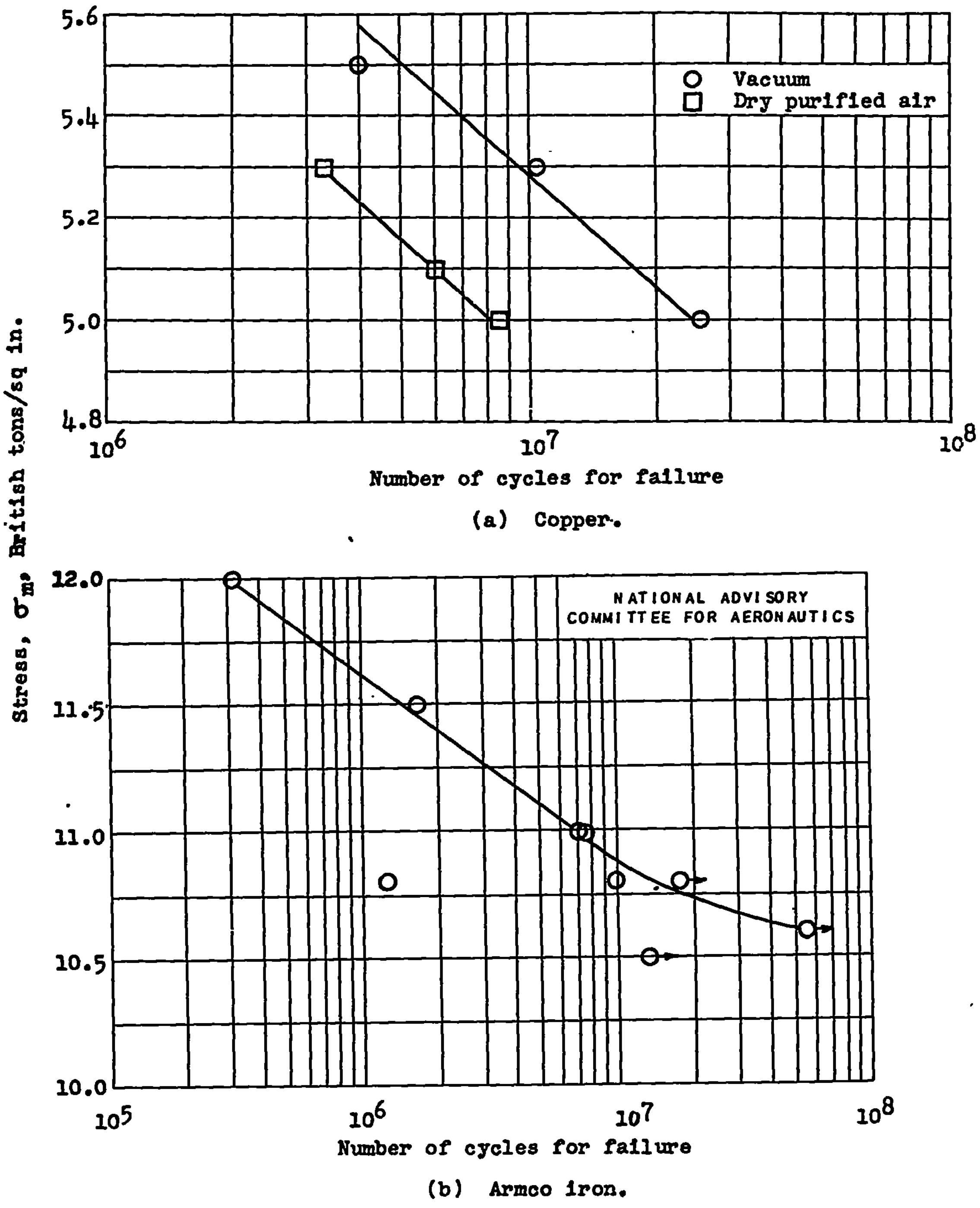

Figure 4. - Variation of stress with number of cycles for failure at room temperature. Frequency, 2200 cycles per minute. (Data taken from reference 12.)

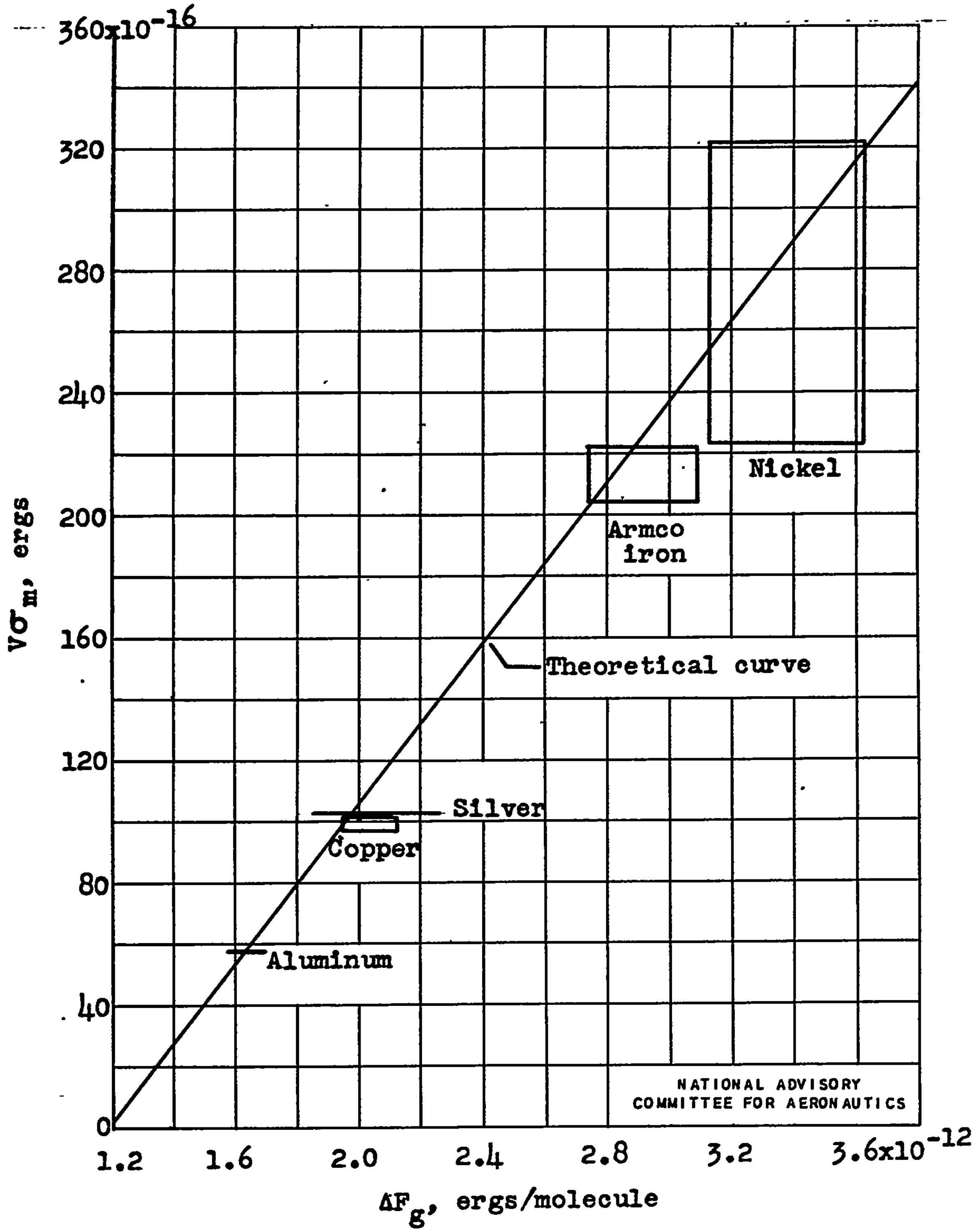

Figure 5. - Dependence of stress corresponding to failure in 3.3×10^6 cycles on material parameters at room temperature.

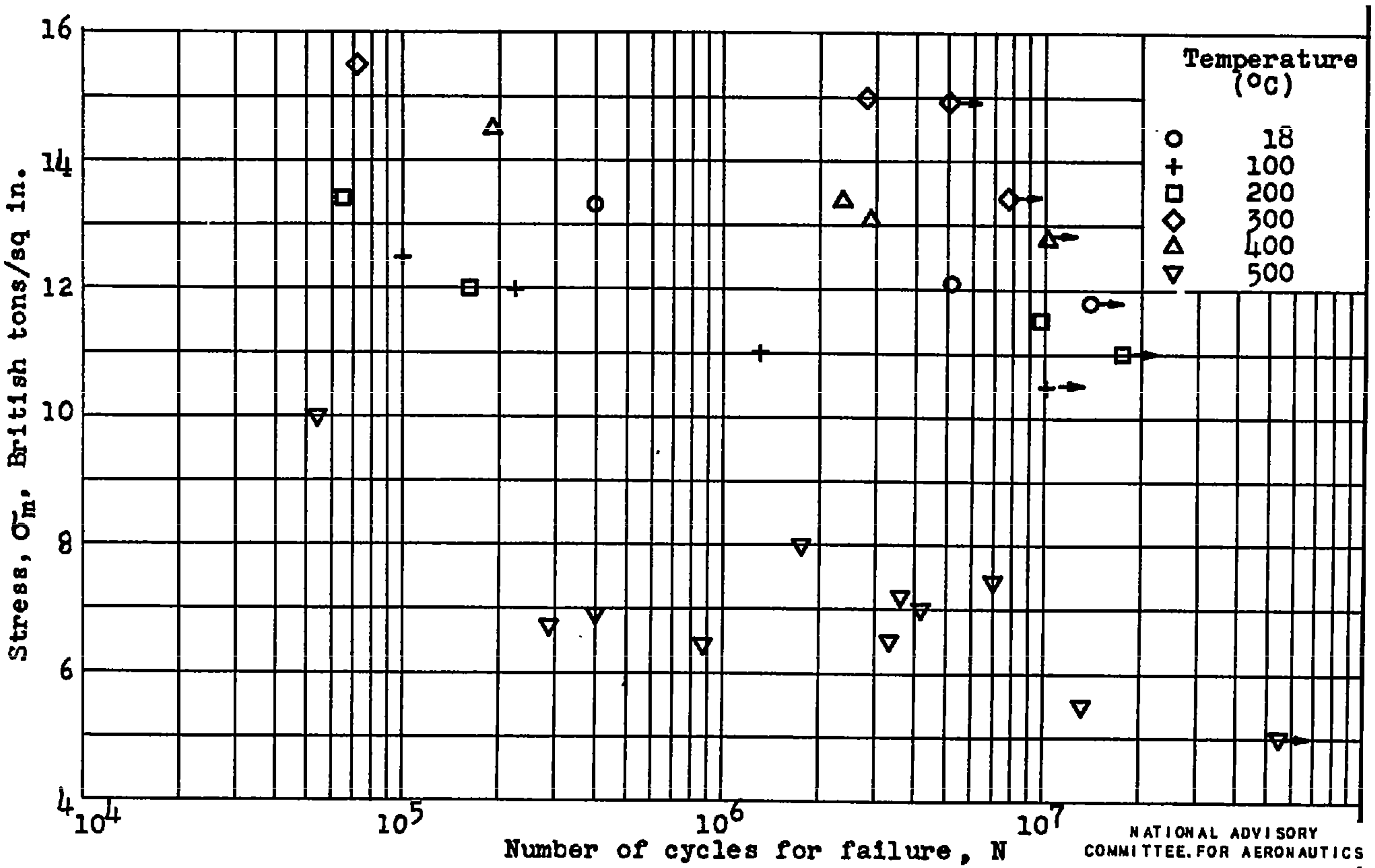

Figure 6. – Temperature dependence of curve for σ_m plotted against N for Armco Iron. (Data taken from reference 14.)

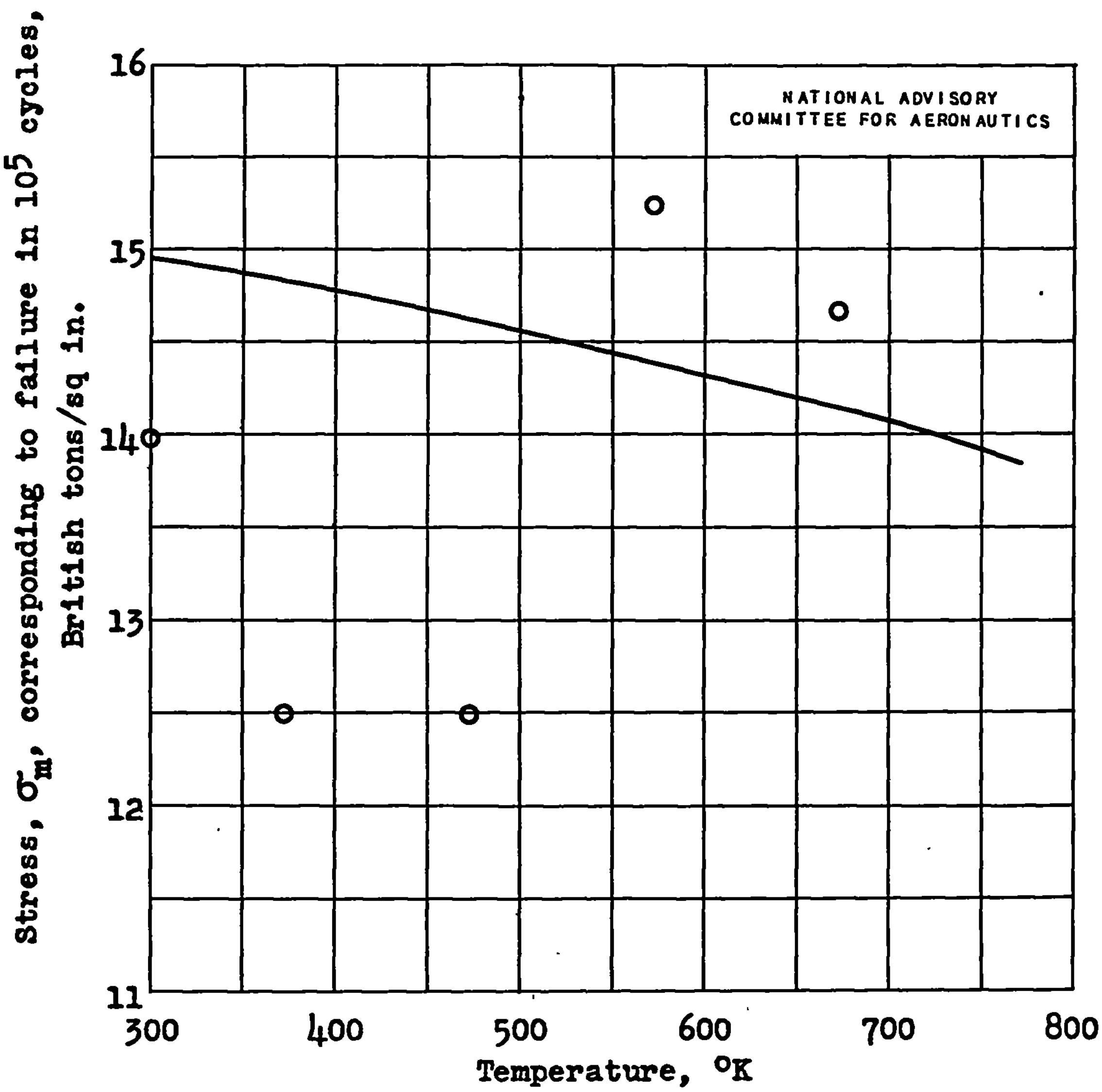

Figure 7. - Temperature dependence of stress corresponding to failure in 10^5 cycles for Armco iron.

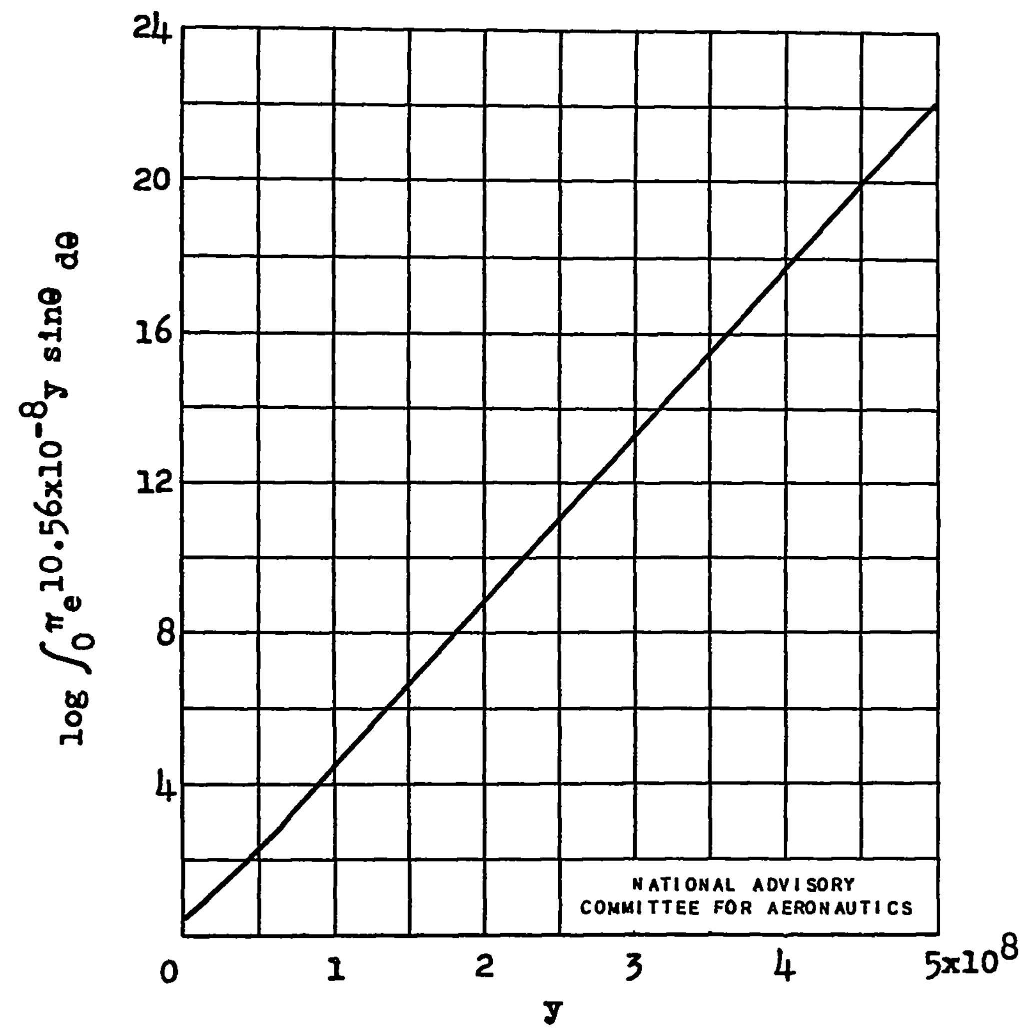

Figure 8. - Variation of $\log\int_0^{\pi} e^{10.56\times10^{-8}y\,\sin\theta}\,d\theta$ with y.

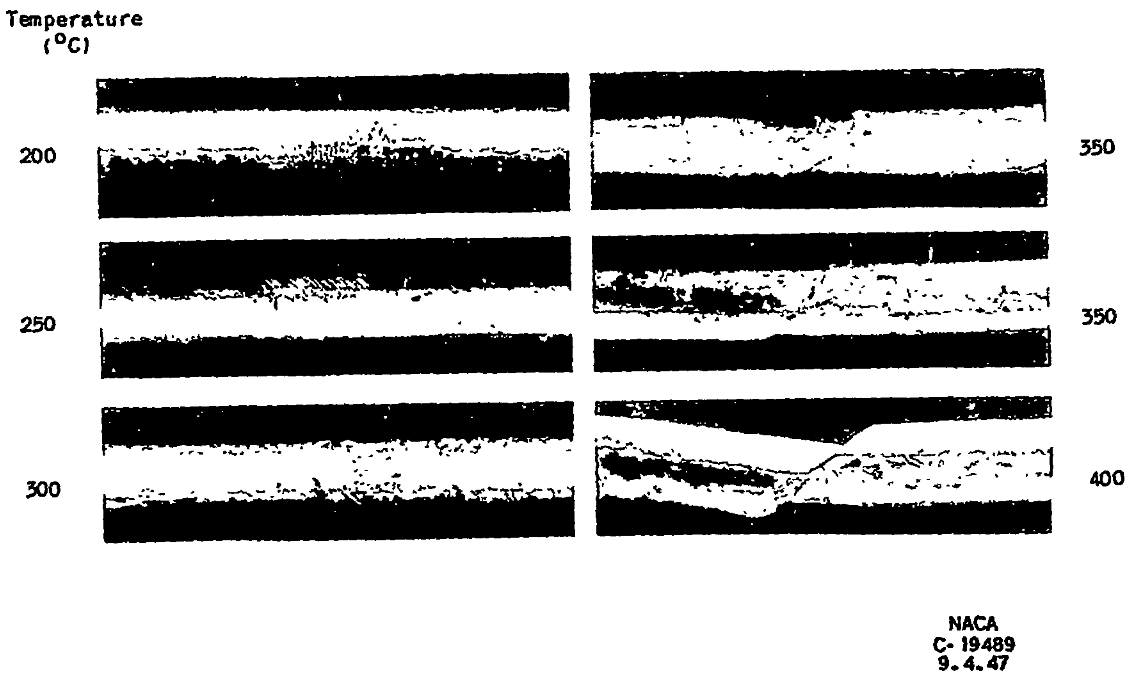

Figure 9. - Pure aluminum sample elongated at different temperatures by application of stress. Most of plastic flow occurs along slip bands. Not etched. X15. (Taken from fig. 14 of reference 18.)

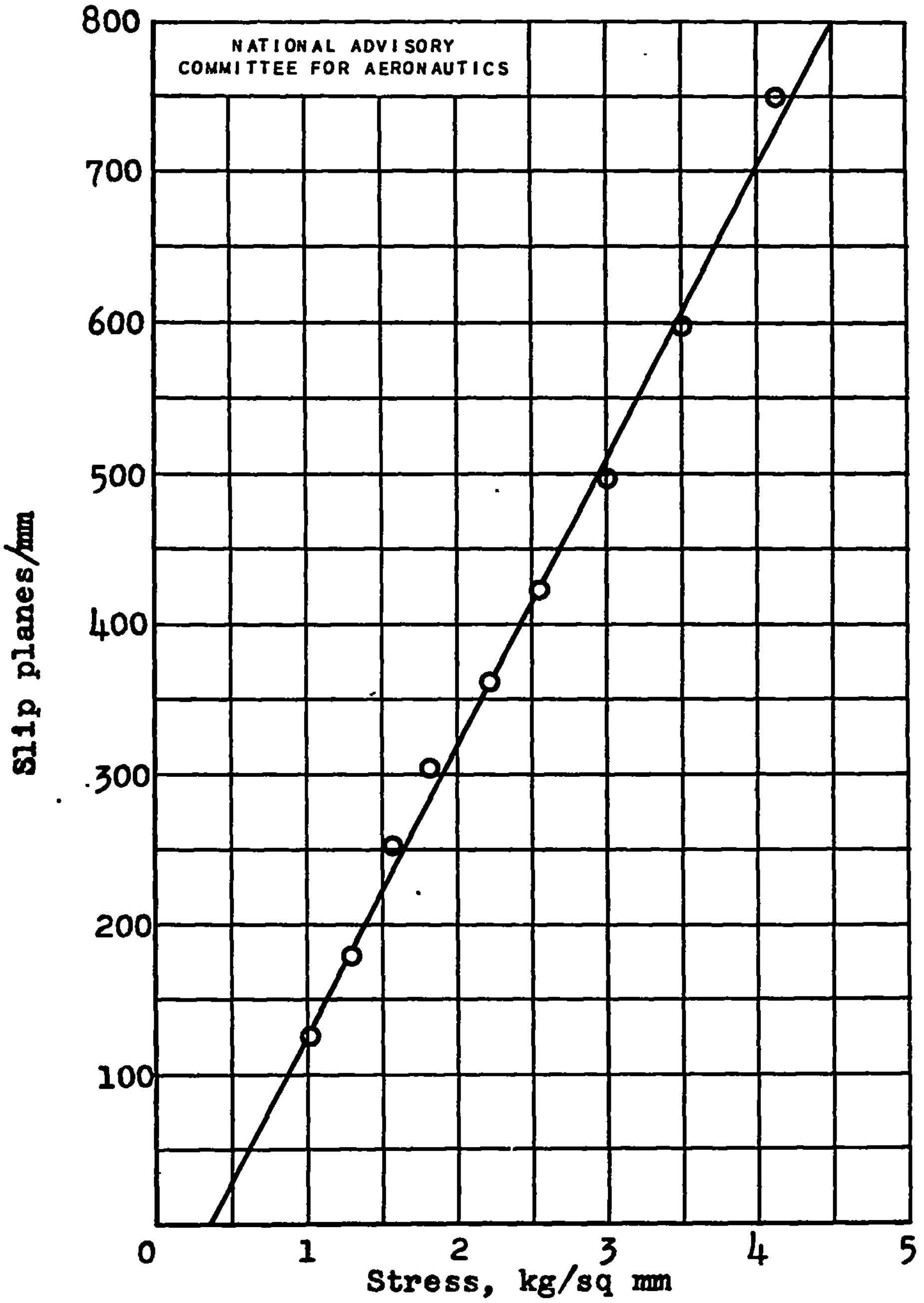

Figure 10. - Stress dependence of number of slip planes per millimeter. (Taken from fig. 24 of reference 19.)